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SECOND-ORDER ELONGATION OF METAL TUBES IN CYCLIC TORSION

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Abstract—Aluminum tubes of different wall-thickness are subjected to reversed cyclic torsion of the same amplitude superimposed on the same axial tensile stress. The observed strong dependence on wall thickness of the axial extension accumulating with the number of torsion cycles ("cyclic creep") implies that this phenomenon is due to the amplification, by superimposed tension, of accumulating second-order strain increments.

1. INTRODUCTION

WHEN a polycrystalline metal bar or tube under axial tension that by itself produces only a small or no permanent elongation, is subjected to cyclic torsion of small amplitude (shear strain $\pm\gamma < 10^{-2}$), it starts to extend rapidly by accumulation of irreversible strain-increments in each half cycle; observations of this phenomenon known as "cyclic creep" have been reported for various metals [1] without convincing explanation. None of the current plasticity theories can predict a steady axial strain-accumulation produced by small cyclic torsion interacting with tension. Tentative mechanisms suggested to account for the "cyclic creep" are

(a) softening, resulting from cyclic-strain-induced recovery processes in the metal, which counteracts the strain-hardening that otherwise blocks the extension [2];

(b) reduction of the yield limit under combined tension and torsion by the Bauschinger effect resulting from reversal of torsion; thus at some point in each cycle the combined stress exceeds the reduced yield limit [3].

On the basis of a recent systematic experimental study of the accumulation of axial strain due to cyclic torsion with and *without* axial tension [4] it was concluded, however, that the strain accumulation could only be the manifestation of an irreversible second-order effect that had not previously been reported, but which represents the logical counterpart, in the strain-hardening medium, of the so-called Poynting effect in the elastic medium. The difference which makes this effect physically so much more significant and also so much more easily observable than the Poynting effect is its irreversibility; it accumulates and can therefore be arbitrarily magnified by repeated strain-cycling, subject only to the limitations of progressive strain-hardening or fatigue cracking [5].

While the assumption that mechanism (a) can account for cyclic strain accumulation has been shown to be untenable, it is possible to base a reasonable explanation of this accumulation on mechanism (b), [6], independently of the fact that the existence of second-order axial strain accumulation produced by cyclic torsion *without* applied tension has been clearly demonstrated, and that it has been shown that the proposed new constitutive equation of the strain-hardening medium provides for the possibility that the axial extension under the combination of tension and cyclic torsion is the result of second-order strain amplified by its interaction with the applied tension, and magnified by the repeated cycling [7]. The persistence into the range of interaction with small tensile stresses of the quadratic relation between the cyclic shear strain and the axial extension, which is characteristic for second-order extension due to torsion without tension, strengthens the belief that second-order strain is a condition for the existence of "cyclic creep." Nevertheless, this evidence, while reasonably convincing, is not conclusive as long as the alternative mechanism (b) is not shown to be contradicted by observations that can be explained only by second-order theory.

2. EXPERIMENTS AND RESULTS

The experiments described in this paper have been designed for this purpose. They are concerned with the observation of the accumulation of axial strain caused by torsion cycling without and with superimposed axial tension, in metal tubes of different wall-thickness. The solution of the problem of torsion of a thick-walled elastic tube on the basis of a constitutive equation containing a second-order term required by the condition of coordinate invariance of the tensor relation leads to the prediction that the normal stress σ_z exerted by the longitudinally restrained cylinder or the resulting longitudinal extension of the unrestrained cylinder is a function of the wall-thickness, and increases with decreasing thickness [8]. For the hollow elastic cylinder of outer radius R and inner radius r the normal force N_z

$$N_z = \text{const.}\ \varphi^2(R^4 - r^4) \tag{1}$$

where the constant depends on a combination of physical constants, φ is the angle of twist and $\delta = (R-r)$ is the wall-thickness. For a solid section $N_{zs} = \text{const.}\ \varphi^2 R^4$, while for a thin-walled tube $(R+r) \sim 2R$ and therefore $N_{zt} \doteq \text{const.}\ \varphi^2 \,.\, 4R^3\delta$. The mean axial stress in the thin-walled tube $\bar{\sigma}_{zt} = \text{const.}\ 2\varphi^2 R^2/\pi$ is therefore twice that in the solid cylinder $\bar{\sigma}_{zs} = \text{const.}\ \varphi^2 R^2/\pi$. So far no solution of this problem exists for the strain-hardening hollow cylinder. Considering, however, that in rough approximation the strain-hardening material in the loading range can be represented by an elastic material with a modulus that decreases with increasing stress, the assumption seems plausible that in such a material the form of equation (1) would be preserved. However if the hollow cylinder is longitudinally unrestrained the axial extension must be expected to increase faster than the stress, so that the ratio of the accumulating second-order strain-increments in the thin-walled tube and the solid cylinder may be considerably higher than the ratio of the stresses.

If tubes of different wall-thickness are subjected to the *same* constant tensile stress on which reversed cyclic torsion of the same amplitude is superimposed, the resulting accumulating axial strain ("cyclic creep") must be expected to retain the strong dependence on wall-thickness that is characteristic of second-order strain accumulation without tension, if the accumulating strain is due to the amplification of this second-order strain by the interacting tension. If, on the other hand, it is a first-order plastic strain increment produced by the cyclic reduction, due to the Bauschinger effect, of the yield limit below the level of the combination of tensile and shear stress, its magnitude should be independent of wall-thickness, provided the applied tensile stress in tubes of different wall-thickness is kept constant by reducing the tensile force in proportion to the area of the cross-section.

Tubes of 99·99% aluminum were used in the experiments. Their outer diameter was $D = 0{\cdot}25$ in., while variation of the inner diameter produced a series of tubes of 2, 3, 4, 5, 6 and 8×10^{-2} in. wall-thickness; the series included a solid specimen; the tubes were bored out of solid bars. The torque-twist diagram obtained on the tube with the smallest wall-thickness is shown in Fig. 1.

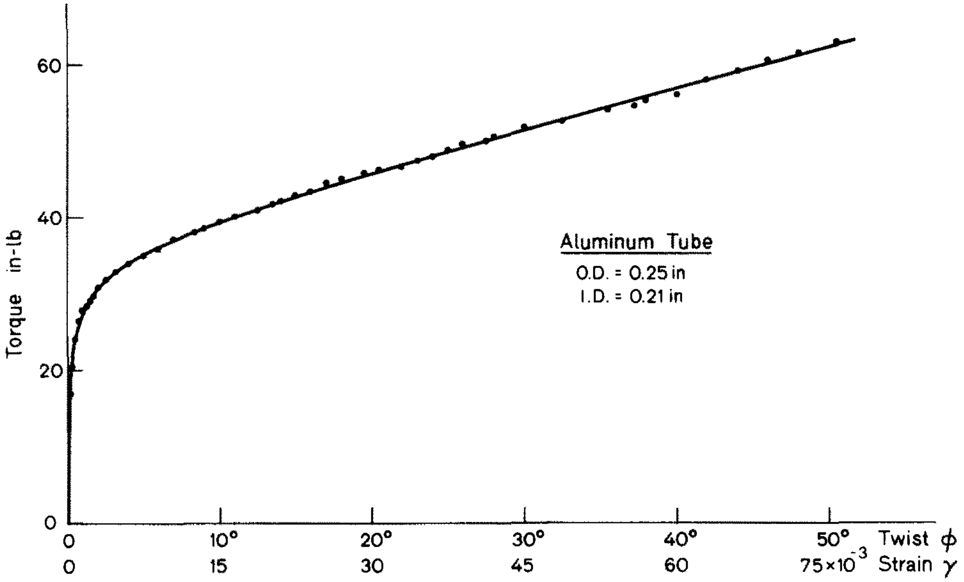

FIG. 1. Uni-directional torque-twist diagram obtained on aluminum tube.

The specimens were mechanically polished and subsequently annealed *in vacuo* for 5 hr at 600–660°F; the average grain size after the anneal was 0·1 mm. Figure 2 shows the size of grains in radial direction of the tube with the smallest (2×10^{-2} in.) wall-thickness. The number of grains across the wall is large enough to justify the consideration of the material as quasi-isotropic, a fact which is important for the validity of the theory of second-order effects.

In the alternating torsion fatigue machine used in the tests and operating at 1750 cpm, the imposed total strain amplitude (plastic and elastic) can be varied by means of a variable eccentric [9]. This twist is applied to one end of a specimen; the other end cannot rotate but is free to slide in longitudinal direction. The axial strain accumulating during alternating torsion is measured by means of a micrometer attached to the frame of the machine and making contact with a screw attached to the sliding grips. Extensions can thus be determined as a function of the number of torsion cycles. The reversed torsion can be applied with or without an additional axial load. In both cases axial extension of the specimen is continuously measured during the experiments. With a gage length of 1·375 in. a twist of $\varphi = \pm 1°$ produces $\gamma = \pm 15 \times 10^{-4}$ reversed shear strain in the tubes.

In an elastic medium the small longitudinal compressive force N_z can prevent the reversible second-order extension produced by an applied torque [10]. In a work-hardening medium the application of such a force hinders the accumulation of irreversible second-order extension produced by cyclic torsion [11]; as a consequence the specimen starts to buckle after a number of torsion cycles which depends on wall-thickness and amplitude of cyclic strain. It has been shown, however, that the compressive stress that could restrain the axial extension is so small that the effect of friction in the sliding parts of the machine cannot be neglected even though these parts were designed for minimum friction.

This friction was partly overcome by the application of a 10 g axial load in one series of tests and of a 12·5 g axial load in the other series of tests. These axial loads did not produce strain in the specimen that could be detected in measurements with 1×10^{-6} in. accuracy. In fact these loads still appeared to leave a very small compressive force acting against the axial extension of the specimen, since the thinnest tubes buckled slightly in the course of the axial extension as marked on Fig. 3 in which the test results are presented for

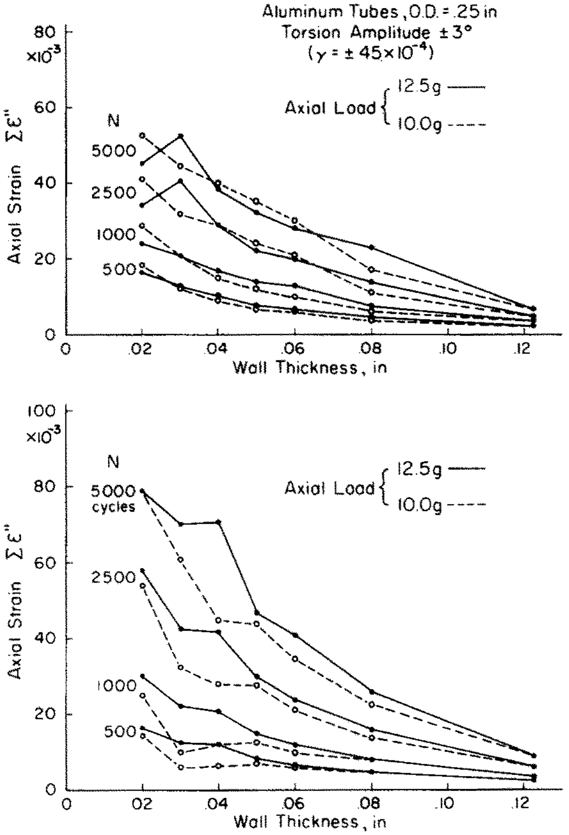

FIG. 3. Effect of wall-thickness of aluminum tubes on accumulating second-order axial strain under N cycles of reversed torsion. Test series 1 and 2 ($\varphi = \pm 3°$).

different numbers of torsion cycles. The reason for not increasing the axial load till buckling was completely eliminated was the wish to avoid the application to the specimen, of even the slightest axial tension. The axial strains recorded in the series of tests conducted with 10 and 12·5 g axial load can thus be considered as very slightly restrained pure second-order extensions.

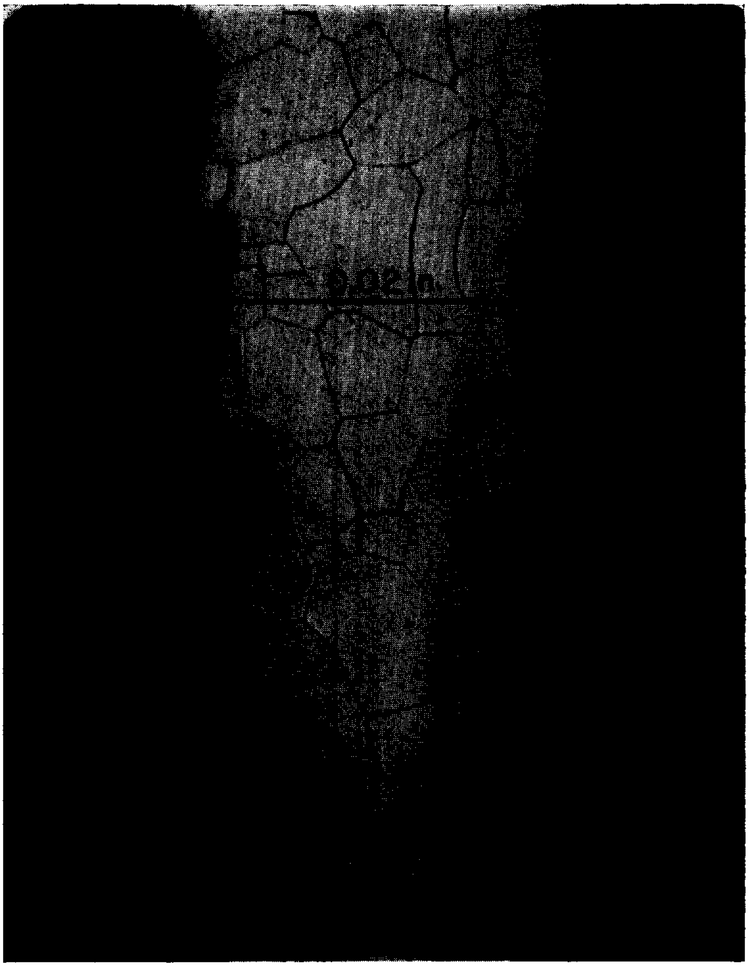

FIG. 2. Grain structure across wall-thickness of tube.

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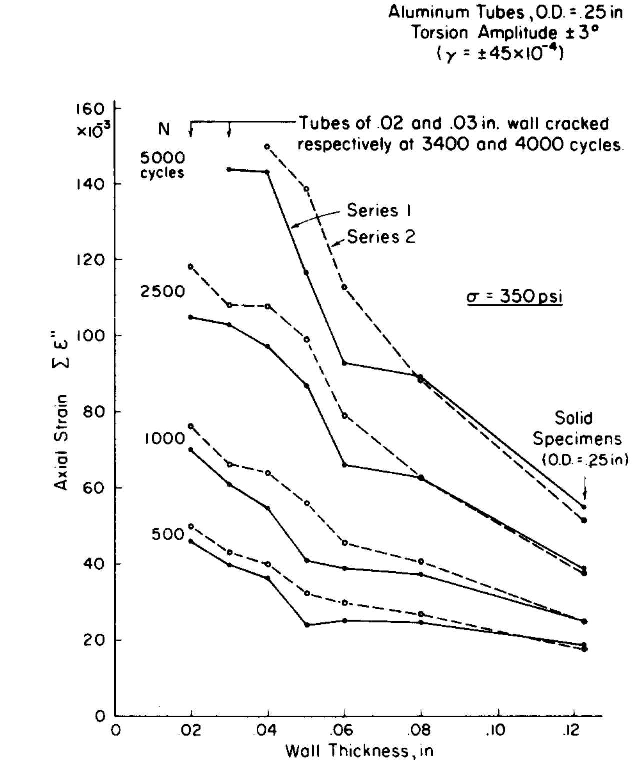

Fig. 4. Effect of wall thickness of aluminum tubes on axial strain under a constant axial tensile stress $\sigma = 350$ psi with superimposed N cycles of reversed torsion ($\varphi = \pm 3°$).

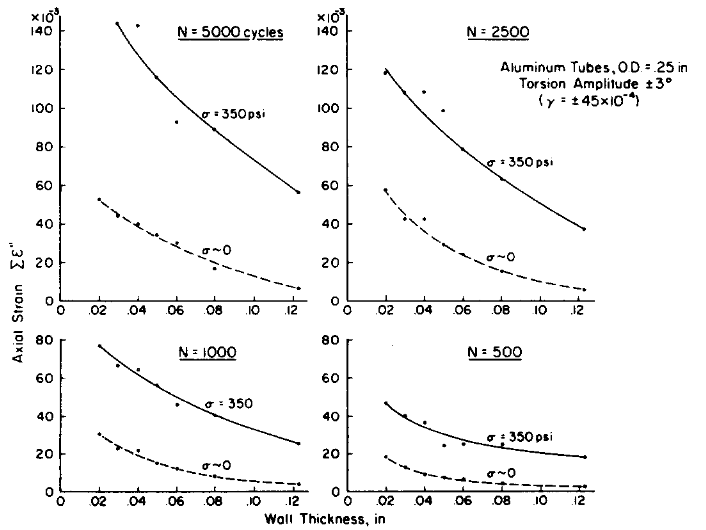

Fig. 5. Comparison between the effects of wall thickness on axial strain accumulation under N cycles of reversed torsion with and without axial tensile stress ($\varphi = \pm 3°$).

In another test series of tubes of different wall-thickness the same strain amplitude in reversed cyclic torsion was superimposed on axial tensile forces that produced in all tubes the same tensile stress of 350 psi. The result of these tests are presented in Fig. 4. They show conclusively that the effect of the wall-thickness characteristic of the pure second-order extension remains practically unchanged when an axial stress is applied. This conclusion is confirmed by Fig. 5 in which the results of tests with and without axial tension are compared for different numbers of load cycles.

The effect of wall-thickness almost vanishes, however, when the applied cyclic torsion amplitude is substantially increased from $\varphi = \pm 3°$ or $\gamma = 4{\cdot}5 \times 10^{-4}$, to $\varphi = 20°$ or $\gamma = 3 \times 10^{-2}$, as demonstrated by the test results presented in Fig. 6(a). This is probably

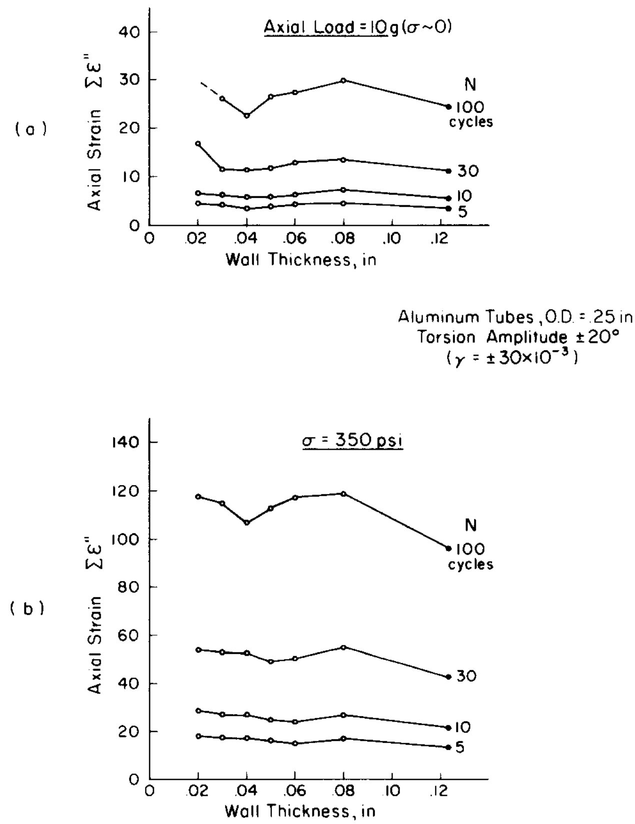

FIG. 6. Effect of wall thickness on axial strain accumulation under N cycles of reversed torsion $\varphi = \pm 20°$ (a) without and (b) with $\sigma = 350$ psi axial tensile stress.

the result of the known [12] reduction of second-order extension at strains beyond one or two percent, which is closely related to the flattening of the stress–strain diagram in this region (see Fig. 1). However, the superposition of a tensile stress of 350 psi reproduces almost exactly, at a higher level of axial strain, the diagrams obtained in pure cyclic torsion [Fig. 6(b)]. This is clearly shown in Fig. 7 where the axial extension for $N = 100$ due to cyclic torsion with $\sigma = 0$ and $\sigma = 350$ psi are compared.

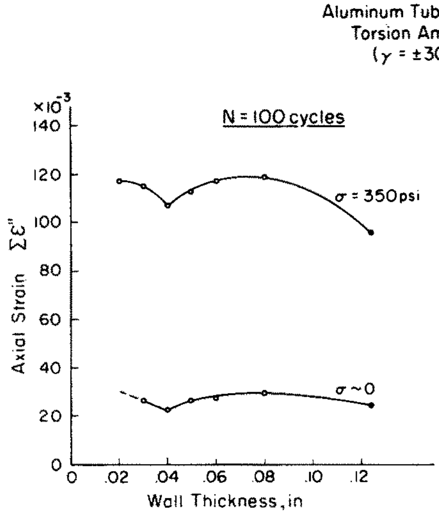

FIG. 7. Comparison between diagrams for $N = 100$ cycles of Figs. 6(a) and (b).

3. CONCLUSIONS

The results of the cyclic torsion experiments on metal tubes of different wall-thickness provide the evidence that the mechanism of axial strain accumulation under combined tension and cyclic torsion ("cyclic creep") is the amplification, by second-order interaction, of the second-order strain accumulation produced by cyclic torsion alone. No other suggested mechanism can provide an explanation for the effect of the wall-thickness of a tube on the axial extension under the same tensile stress.

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Абстракт—Алюминиевые трубы с разной толщиной стенки, подверженые обратному циклическому кручении с такойже самой амплитудой, накладываемой на осевое напряжение растяжения. Наблюдаемая сильная зависимость на толщину стенки осевого удлиннения, аккуммулируюего с числом циклов кручения /"циклическая ползучесть"/ вызывает, что это явление влияет на увеличение, путем накладываемого растяжения, аккумулирующих приращений деформации второго ранга.